Modular Representation Theory of Finite Groups

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Foreword

This text constitutes a faithful transcript of the lecture **Modular Representation Theory** held at the TU Kaiserslautern during the Winter Semester 2022/23 (14 Weeks, 4SWS Lecture + 2SWS Exercises).

Together with the necessary theoretical foundations the main aims of this lecture are to:

- provide students with a modern approach to **finite group theory**;
- learn about the **representation theory of finite-dimensional algebras** and in particular of **the group algebra of a finite group**;
- establish connections between the representation theory of a finite group over a field of **positive** characteristic and that over a field of characteristic **zero**;
- consistently work with **universal properties** and get acquainted with the **language of category theory**.

We assume as pre-requisites bachelor-level algebra courses dealing with *linear algebra* and *elementary group theory*, such as the standard lectures *Grundlagen der Mathematik*, *Algebraische Strukturen*, and *Einführung in die Algebra*. It is also strongly recommended to have attended the lectures *Commutative Algebra* and *Character Theory of Finite Groups* prior to this lecture. Therefore, in order to complement these pre-requisites, but avoid repetitions, the Appendix deals formally with some background material on module theory, but proofs are omitted.
The main results of the lecture *Character Theory of Finite Groups* will be recovered through a different and more general approach, thus it is formally not necessary to have attended this lecture already, but it definitely brings you some intuition.

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Conventions

Unless otherwise stated, throughout these notes we make the following general assumptions:

- all groups considered are **finite**;
- all rings considered are **associative** and **unital** (i.e. possess a neutral element for the multiplication, denoted 1);
- all modules considered are **left** modules;
- if K is a commutative ring and G a finite group, then all KG-modules considered are assumed to be free of finite rank when regarded as K-modules.