• Gabelian

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Conjugacy classes :

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Write $\operatorname{Trr}(G) = \{\chi_{1_1}, \dots, \chi_{n_n}\}$.
Each $\chi_i : G \longrightarrow \mathbb{C}^{\times}$ is a group homomorphism,
hence determined by $\chi_i(g)$, which is an n-th root
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Example: the character table of Cn G := < g | gⁿ = 1> - cyclic group of order ne Z>0. · Characters: Let J be a primitive n-th root of 1c. Write $Irr(G) = \{\chi_1, \dots, \chi_n\}$. Each $\chi_i: G \longrightarrow \mathbb{C}^{\times}$ is a group homomorphism, hence determined by $\chi_i(g)$, which is an n-th root of 1c.

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We obtain

$$X(\zeta_n) = (\chi_i(q_j))_{\substack{1 \le i \le n \\ 1 \le j \le n}} = (\chi_i(q^{j-i}))_{\substack{1 \le i \le n \\ 1 \le j \le n}} = (\Im^{(i-i)(j-i)})_{\substack{1 \le i \le n \\ 1 \le j \le n}}$$

As 'table':		1	19-	g² •••	g ^{h-1}	_
	X4=76	1	1	1	. 1	-
	X2	1	3	y ²	- y ⁿ⁻¹	
	X ₃	1	y²	ያዛ	g ^{2(n−1)}	
		• •	e •		: ge-1) ²	
	Xn	1	Y ⁿ⁻¹	y ²⁽ⁿ⁻¹)	Je-I)	

 $G := S_3$

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and $|C_1| = 1$, $|C_2| = 3$, $|C_3| = 2$

Next, we calculate the characters of G and their values.

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$$\begin{split} f_{4}: \ S_{3} \longrightarrow \mathbb{C} \\ & \sigma \longmapsto 1 \\ \\ f_{2}: \ S_{3} \longrightarrow \mathbb{C}^{\times} \\ & \sigma \longmapsto \operatorname{sgn}(\sigma) \\ \\ f_{3}: \ S_{3} \longrightarrow \mathbb{GL}_{2}(\mathbb{C} \\ & (12) \longmapsto \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ & (123) \longmapsto \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} \end{split}$$

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$$P_{4}: S_{3} \longrightarrow C^{*}$$

$$\sigma \mapsto 1$$

$$m > \chi_{4}(\sigma) = 1 \quad \forall \sigma \in S_{3}$$

$$\begin{split} f_{2}: S_{3} \longrightarrow \mathbb{C}^{\times} \\ \sigma \longmapsto sgn(\sigma) \\ f_{3}: S_{3} \longrightarrow GL_{2}(\mathbb{C}) \\ (12) \longmapsto \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ (123) \longmapsto \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} \end{split}$$

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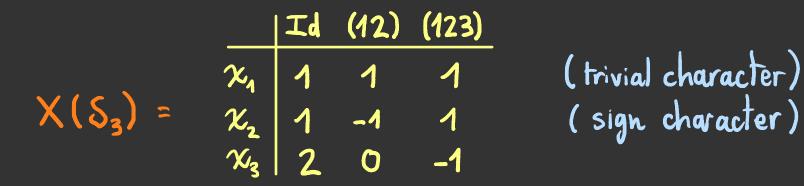
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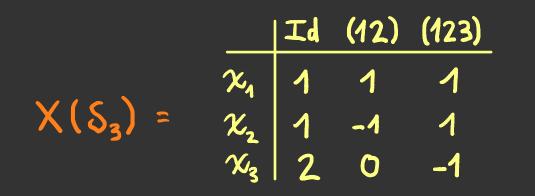
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Notice: the degree formula reds
$$\chi_1(1)^2 + \chi_2(1)^2 + \chi_3(1)^2 = 1 + 1 + 4 = 6 = |G|$$

=> The character table of S_3 is :

$$\begin{array}{c|c} & \text{Id} & (12) & (123) \\ \hline \chi_1 & 1 & 1 & 1 \\ \chi(S_3) = & \chi_2 & 1 & -1 & 1 \\ \chi_3 & 2 & 0 & -1 \end{array}$$

Notice: the degree formula reds $\chi_1(4)^2 + \chi_2(4)^2 + \chi_3(4)^2 = 4 + 4 + 4 = 6 = 161$ so, we could also have deduced from this that χ_4, χ_2, χ_3 are all the irreducible characters of S₃.