COHOMOLOGY OF GROUPS — EXERCISE SHEET 9 | **TU KAISERSLAUTERN**

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EXERCISE 1

Let $1 \longrightarrow A \xrightarrow{i} E \xrightarrow{p} G \longrightarrow 1$ be a group extension. Prove that TFAE:

(a) i has a group-theoretic retraction;

(b) A has a normal complement in E;

(c) there is a subgroup H of E such that $E \cong A \times H$.

Exhibit examples of split extensions of groups which do not admit a group-theoretic retraction.

EXERCISE 2

Let $1 \longrightarrow A \xrightarrow{i} E \xrightarrow{p} G \longrightarrow 1$ be a group extension.

(a) Prove that if G is a free group, then the extension splits.

(b) Prove that any group homomorphism $E \longrightarrow E$ inducing the identity on A and on G is an isomorphism.

EXERCISE 3

(a) Consider the dihedral 2-group D_{2^n}, $n \geqslant 3$, and A its cyclic subgroup of index 2. How many G-conjugacy classes of complements in G are there? Describe them all.

(b) Same question for $E = (\underbrace{A \times \cdots \times A}_{m \text{ factors}}) \rtimes C_m$, where A is an abelian group and C_m acts by cyclic permutation.

EXERCISE 4

Let G be a finite group, let A be a finite trivial $\mathbb{Z}G$-module, and assume that $(|A|, |G|) = 1$.

(a) Prove that $H^1(G, A) = 0$.

(b) Find all complements of A in $A \times G$.