COHOMOLOGY OF GROUPS — EXERCISE SHEET 4 BIS **TU KAISERSLAUTERN**

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Throughout these exercises R denotes an associative and unital ring.

EXERCISE 1

Let $\mathbf{C}_\bullet$ be a chain complex of R-modules. Prove that TFAE:

(a) $\mathbf{C}_\bullet$ is exact, i.e. exact at C_n for each $n \in \mathbb{Z}$;

(b) $\mathbf{C}_\bullet$ is acyclic, i.e. $H_n(\mathbf{C}_\bullet) = 0$ for all $n \in \mathbb{Z}$;

(c) The chain map $\mathbf{0}_\bullet \longrightarrow \mathbf{C}_\bullet$ is a quasi-isomorphism.

EXERCISE 2

Formulate the following definitions formally:

· of a subcomplex and of a quotient complex of a cochain complex of R-modules;

· of kernels, images, and cokernels of morphisms of cochain complexes of R-modules.