Cohomology of Groups — Exercise Sheet 4 | **TU Kaiserslautern**

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Throughout these exercises R denotes an associative, unital ring.

Exercise 1
Verify that for each $n \in \mathbb{Z}$, $H_n(-) : \mathbf{Ch}(_R\mathbf{Mod}) \longrightarrow {}_R\mathbf{Mod}$ is a covariant functor.

Exercise 2

(a) Let p be a prime number and consider the following chain complexes of $\mathbb{Z}$-modules:

$$\cdots \to 0 \to \mathbb{Z} \xrightarrow{\cdot p} \mathbb{Z} \to 0 \to \cdots$$

$$\cdots \to 0 \to \mathbb{Z} \xrightarrow{0} \mathbb{Z} \to 0 \to \cdots$$

$$\cdots \to 0 \to \mathbb{Z}/2\mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z}/4\mathbb{Z} \to 0 \to \cdots$$

$$\cdots \to 0 \to \mathbb{Z}/3\mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z}/6\mathbb{Z} \to 0 \to \cdots$$

Compute the homology of each complex.

(b) Consider the following morphism of chain complexes of abelian groups:

$$\begin{array}{ccccccccccccccc}
\cdots & \xrightarrow{0} & \mathbb{Z} & \xrightarrow{=} & \mathbb{Z} & \xrightarrow{0} & \mathbb{Z} & \xrightarrow{=} & \mathbb{Z} & \xrightarrow{0} & \mathbb{Z} & \xrightarrow{\cdot p} & \mathbb{Z} & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow{\scriptstyle \pi} & & \\
\cdots & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & \mathbb{Z}/p\mathbb{Z} & \longrightarrow & 0
\end{array}$$

Compute the homology of both the horizontal chain complexes and prove that each map induced in homology by the vertical maps is an isomorphism.