COHOMOLOGY OF GROUPS — EXERCISE SHEET 4

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Throughout these exercises *R* denotes an associative, unital ring.

Exercise 1

Verify that for each *n* ∈ \mathbb{Z} , $H_n(-)$: **Ch**($_R$ **Mod**) $\longrightarrow _R$ **Mod** is a covariant functor.

Exercise 2

(a) Let p be a prime number and consider the following chain complexes of \mathbb{Z} -modules:

$$\cdots \to 0 \to \mathbb{Z} \xrightarrow{\cdot p} \mathbb{Z} \to 0 \to \cdots$$

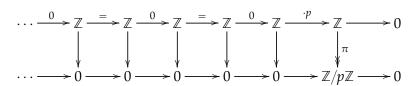
$$\cdots \to 0 \to \mathbb{Z} \xrightarrow{0} \mathbb{Z} \to 0 \to \cdots$$

$$\cdots \to 0 \to \mathbb{Z}/2\mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z}/4\mathbb{Z} \to 0 \to \cdots$$

$$\cdots \to 0 \to \mathbb{Z}/3\mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z}/6\mathbb{Z} \to 0 \to \cdots$$

Compute the homology of each complex.

(b) Consider the following morphism of chain complexes of abelian groups:



Compute the homology of both the horizontal chain complexes and prove that each map induced in homology by the vertical maps is an isomorphism.