Cohomology of Groups — Exercise Sheet 13 | **TU Kaiserslautern**

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Throughout these exercises all groups are assumed to be given in multiplicative notation, and K denotes a field of characteristic $p \geqslant 0$.

Exercise 1

Let B be an abelian group. Prove that if a central extension $1 \longrightarrow Z \longrightarrow E \xrightarrow{\nu} G \longrightarrow 1$ of groups is B-universal, then the transgression $\mathrm{tr} : \mathrm{Hom}(Z, B) \longrightarrow H^2(G, B)$ is surjective.

Exercise 2

Let G be a finite group and assume that $K = \overline{K}$ is algebraically closed of characteristic $p \geqslant 0$. Prove that any cohomology class $c \in H^2(G, K^\times)$ can be represented by a 2-cocycle $\alpha : G \times G \longrightarrow K^\times$ whose values are $o(c)$-th roots of unity in K.

Exercise 3

Assume $K = \mathbb{C}$ and let $1 \longrightarrow Z \longrightarrow E \xrightarrow{\pi} G \longrightarrow 1$ be a central extension of finite groups. Prove that if $Z \leqslant [E, E]$, then the transgression $\mathrm{tr} : \mathrm{Hom}(Z, \mathbb{C}^\times) \longrightarrow H^2(G, \mathbb{C}^\times) = M(G)$ is injective.