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Due date: **Monday, 5th of July 2021, 18:00** | SS 2021

Exercise 1

Let p be a prime number, let G be a finite group of order divisible by p, and let P be a Sylow p-subgroup of G. If A is an $\mathbb{F}_pG$-module, then the restriction map

$$\operatorname{res}^G_P : H^n(G, A) \longrightarrow H^n(P, \operatorname{Res}^G_P(A))$$

is injective for every $n \geqslant 0$.

Exercise 2

Let p be a prime number. Let G be a finite group of oder divisible by p and let P be a Sylow p-subgroup of G.

(a) Prove that $C_G(P) = Z(P) \times H$, where H is a subgroup of G of order coprime to p.

(b) Prove that $PC_G(P) = P \times H$.

(c) Prove that $N_G(P) = P \rtimes K$, where K is a subgroup of G of order coprime to p, and $H \trianglelefteq K$.

Exercise 3

Let G be a finite group of even order and assume that a Sylow 2-subgroup P of G is cyclic.

(a) Prove that $\operatorname{Aut}(P)$ is a 2-group.

(b) Prove that P has a normal complement in G.