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Due date: **Monday, 28th of June 2021, 18:00** | SS 2021

Exercise 1
Let A be a $\mathbb{Z}G$-module, written multiplicatively and let $f : G \times G \longrightarrow A$ be a normalised 2-cocycle. Let $E_f = A \times G$ with product

$$(a, g)(b, h) = (a\, {}^g b f(g, h), gh) \qquad \forall\, (a, g), (b, h) \in E_f.$$

Using the 2-cocycle identity, prove that E_f is a group and that the left and right inverses coincide, that is:

$$({}^{g^{-1}}a^{-1} f(g^{-1}, g)^{-1}, g^{-1}) = ({}^{g^{-1}}a^{-1}\, {}^{g^{-1}}f(g, g^{-1})^{-1}, g^{-1}) \quad \forall\, (a, g) \in E_f.$$

Moreover, verify that there is an extension $1 \longrightarrow A \longrightarrow E_f \longrightarrow G \longrightarrow 1$ associated with the 2-cocycle f which induces the given G-action on A.

Exercise 2

(a) Let $A := C_4$ and $G := C_2$.

- Find all actions by group automorphisms of G on A.
- For each such action, compute $H^2(G, A)$.
- In each case, describe all extensions of A by G inducing the given action, up to equivalence.

(b) Let $G := C_2 \times C_2$ and $A := C_2$ regarded as a trivial $\mathbb{Z}G$-module. Assume known that $H^2(G, A) \cong (\mathbb{Z}/2)^3$.

- Given $1 \longrightarrow A \longrightarrow E \longrightarrow G \longrightarrow 1$ an arbitrary central extension of A by G, determine a presentation of the group E using a presentation of A and a presentation of G.
- Find all central extensions of A by G, up to equivalence, using the previous point.

(c) Classify groups of order 8 up to isomorphism.