-1

 $E(8) - E(8)^3$

 $-E(8) + E(8)^3$

 $E(16) - E(16)^7$

2 0

 $-2 \quad 0 \quad E(8) - E(8)^3$

The group G is isomorphic to the group labelled by [32, 20] in the Small Groups library. Ordinary character table of $G \cong Q32$:

Trivial source character table of $G \cong Q32$ at p = 2:

| Normalisers N_i | N_1 | N_2 | N_3 | N_4 | N_5 | N_6 | N_7 | N_8 | N_9 | N_{10} | N_{11} | N_{12} |
|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|
| p-subgroups of G up to conjugacy in G | P_1 | P_2 | P_3 | P_4 | P_5 | P_6 | P_7 | P_8 | P_9 | P_{10} | P_{11} | P_{12} |
| Representatives $n_j \in N_i$ | 1a | 1a | 1a |
| $1 \cdot \chi_1 + 1 \cdot \chi_2 + 1 \cdot \chi_3 + 1 \cdot \chi_4 + 2 \cdot \chi_5 + 2 \cdot \chi_6 + 2 \cdot \chi_7 + 2 \cdot \chi_8 + 2 \cdot \chi_9 + 2 \cdot \chi_{10} + 2 \cdot \chi_{11}$ | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $1 \cdot \chi_1 + 1 \cdot \chi_2 + 1 \cdot \chi_3 + 1 \cdot \chi_4 + 2 \cdot \chi_5 + 2 \cdot \chi_6 + 2 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11}$ | 16 | 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\begin{bmatrix} 1 \cdot \chi_1 + 1 \cdot \chi_2 + 1 \cdot \chi_3 + 1 \cdot \chi_4 + 2 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11} \end{bmatrix}$ | 8 | 8 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\begin{bmatrix} 1 \cdot \chi_1 + 1 \cdot \chi_2 + 0 \cdot \chi_3 + 0 \cdot \chi_4 + 1 \cdot \chi_5 + 1 \cdot \chi_6 + 1 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11} \end{bmatrix}$ | 8 | 8 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\begin{bmatrix} 1 \cdot \chi_1 + 0 \cdot \chi_2 + 1 \cdot \chi_3 + 0 \cdot \chi_4 + 1 \cdot \chi_5 + 1 \cdot \chi_6 + 1 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11} \end{bmatrix}$ | 8 | 8 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\begin{bmatrix} 1 \cdot \chi_1 + 1 \cdot \chi_2 + 1 \cdot \chi_3 + 1 \cdot \chi_4 + 0 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11} \end{bmatrix}$ | 4 | 4 | 4 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\begin{bmatrix} 1 \cdot \chi_1 + 1 \cdot \chi_2 + 0 \cdot \chi_3 + 0 \cdot \chi_4 + 1 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11} \end{bmatrix}$ | 4 | 4 | 4 | 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
| $1 \cdot \chi_1 + 0 \cdot \chi_2 + 1 \cdot \chi_3 + 0 \cdot \chi_4 + 1 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11}$ | 4 | 4 | 4 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| $1 \cdot \chi_1 + 1 \cdot \chi_2 + 0 \cdot \chi_3 + 0 \cdot \chi_4 + 0 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11}$ | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 0 | 0 | 0 |
| $1 \cdot \chi_1 + 0 \cdot \chi_2 + 1 \cdot \chi_3 + 0 \cdot \chi_4 + 0 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11}$ | 2 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 0 | 2 | 0 | 0 |
| $1 \cdot \chi_1 + 0 \cdot \chi_2 + 0 \cdot \chi_3 + 1 \cdot \chi_4 + 0 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11}$ | 2 | 2 | 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 2 | 0 |
| $\begin{bmatrix} 1 \cdot \chi_1 + 0 \cdot \chi_2 + 0 \cdot \chi_3 + 0 \cdot \chi_4 + 0 \cdot \chi_5 + 0 \cdot \chi_6 + 0 \cdot \chi_7 + 0 \cdot \chi_8 + 0 \cdot \chi_9 + 0 \cdot \chi_{10} + 0 \cdot \chi_{11} \end{bmatrix}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

 $P_1 = Group([()]) \cong 1$

 $P_2 = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(18,29)(20,30)(23,31)(27,32)]) \cong \mathbb{C}_2$

 $P_{3} = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(18,29)(20,30)(23,31)(27,32),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32)]) \cong C4$ $P_4 = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(18,29)(20,30)(23,31)(27,32), (1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20)]) \cong C4$ $P_{5} = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(18,29)(20,30)(23,31)(27,32), (1,2,6,10)(3,17,13,28)(4,20,15,30)(5,22,16,9)(7,24,19,11)(8,26,21,14)(12,32,25,27)(18,23,29,31)]) \cong C4$ $P_6 = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(18,29)(20,30)(23,31)(27,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,8,22,30,10,21,9,20)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27)]) \cong C8$ $P_{7} = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(13,29)(20,30)(23,31)(27,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(12,12,12)(12,12,12)(12,12,12)(12,12,12)(12,$ $P_8 = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(18,29)(20,30)(23,31)(27,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(11,$

 $N_3 = Group([(1, 5, 6, 16)(2, 9, 10, 22)(3, 12, 13, 25)(4, 14, 15, 26)(7, 18, 19, 29)(8, 20, 21, 30)(11, 23, 24, 31)(17, 27, 28, 32),$ $N_4 = Group([(1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20),(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32)]) \cong Q8$ $N_{5} = Group([(1, 2, 6, 10)(3, 17, 13, 28)(4, 20, 15, 30)(5, 22, 16, 9)(7, 24, 19, 11)(8, 26, 21, 14)(12, 32, 25, 27)(18, 23, 29, 31), (1, 6)(2, 10)(3, 13)(4, 15)(5, 16)(7, 19)(8, 21)(9, 22)(3, 12, 13, 25)(4, 14, 15, 26)(7, 18, 19, 29)(8, 20, 21, 30)(11, 23, 24, 31)(17, 27, 28, 32)]) \cong Q8$ $N_6 = Group([(1,4,16,26,6,15,5,14)(2,8,22,30,10,21,9,20)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,20,21,30)(12,23,25,27)(18,23,29,31),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,23,24,31)(17,27,28,32),(1,2,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(12,23,12,32),(1,2,6,12)(12,23,12,32)(12,23,12,32),(1,2,6,12)(12,23,12,32)(12,23,12,32),(1,2,6,12)(12,23,12,32),(1,$ $N_7 = Group([(1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32),(1,6)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32),(1,6)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32),(1,6)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32),(1,6)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32),(1,6)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32),(1,6)(2,9,10,22)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27)]) \\ \cong Q16$ $N_8 = Group([(1,2,6,10)(3,17,13,28)(4,20,15,30)(5,22,16,9)(7,24,19,11)(8,26,21,14)(12,32,25,27)(18,23,29,31),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32),(1,6)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(2,30)(23,31)(27,32),(1,4,16,26,6,15,5,14)(2,9,22,30,10,21,9,20)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27)] \\ \simeq Q16$ $N_{9} = Group([(1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20)(1,4,16,26,6,15,5,14)(2,9,10,22)(1,24,19,11)(8,26,21,14)(12,32,25,27)(18,29,22,30)(1,24,19,11)(8,26,21,14)(12,32,25,27)(18,29,22,30)(1,24,19,11)(8,26,21,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22,18)(11,26,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,32,25,27)(18,29,22)(11,24,12,23)(11,25,24,14)(12,25,14)(12,25,14$ $N_{10} = Group([(1,2,6,10)(3,17,13,28)(4,20,15,30)(5,22,16,9)(7,24,19,11)(8,26,21,14)(12,32,25,27)(18,23,29,31)(17,27,28,32)(1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20)]) \\ \cong Q32(3,1)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20)] \\ \cong Q32(3,1)(2,1,2,23)(1,2,2,13,23)(11,22,23,13)(11,22,23,13,23)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23,13)(11,22,23$ $N_{11} = Group([(1,17,4,29,16,32,26,19,6,28,15,18,5,27,14,7)(2,24,8,12,22,23,30,3,10,11,21,25,9,31,20,13)(4,15)(5,16)(7,19)(8,20,21,30)(12,32,25,27)(18,29,20)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(3,12,12,13,25)(4,14,15,26)(7,18,19,29)(3,12,13,25)(4,14,15,26)(7,18,19,29)(3,12,12,13,25)(4,14,15,26)(7,18,19,29)(3,12,12,13,25)(4,14,15,26)(7,18,19,29)(3,12,12,13,25)(4,14,15,26)(7,18,19,29)(3,12,12,13,25)(4,14,15,26)(7,18,19,29)(3,12,12,12,13)(3,12,12,12,13)(3,12,12,12,13)(3,12,12,12,13)(3,12,12,12,12)(3,12,12,12,12)(3,12,12,12)(3,12,12,12)(3,12,12,12)(3,12,12)(3,12,12,12)(3,12,12)(3,12,12)(3,12,12)(3,12,12)(3,12,12)(3,12,12)(3,12,12)(3,12,1$ $N_{12} = Group([(1,2,6,10)(3,17,13,28)(4,20,15,30)(5,22,16,9)(7,24,19,11)(8,26,21,14)(12,32,25,27)(18,23,29,31),(1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(9,29,22,18)(11,26,24,14)(17,30,28,20),(1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(12,23,25,27)(18,23,29,31),(1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(9,29,22,18)(11,26,24,14)(17,30,28,20),(1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19,18,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27),(1,5,6,16)(2,9,10,22)(3,12,13,22)(3,11,25,31,13,24,12,23)(7,12,23,12,13,22)(7,$

 $N_1 = Group([(1,2,6,10)(3,17,13,28)(4,20,15,30)(5,22,16,9)(7,24,19,11)(8,26,21,14)(17,30,28,20)(7,17,29,32,19,28,18,27), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,29,31), (1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20), (1,4,16,26,6,15,5,14)(2,8,22,30,10,21,9,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23,21)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23,21)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23,21)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23,21)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23,21)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23)(2,3,11,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23)(2,3,11,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23)(2,3,11,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23)(2,3,11,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23)(2,3,11,23,23), (1,5,6,16)(2,9,10,22)(3,12,13,23)(4,14,15,26)(7,19)(8,21,23)(2,3,11,23,23), (1,5,6,16)(2,9,12)(3,12,13,23)(4,14,15,26)(7,13,12,13,23)(4,14,15,26)(7,13,12,13,23)(4,14,15,26)(7,13,12,13,23)(4,14,15,26)(7,13,12,13,23)(7,13,23$ $N_2 = Group([(1,2,6,10)(3,17,13,28)(4,20,15,30)(5,22,16,9)(7,24,19,11)(8,26,21,14)(17,30,28,20)(1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(2,3,11,25,31,13,24,12,23)(7,17,29,32,19,28,18,27), (1,5,6,16)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,19)(8,21,23,21,32)(4,14,15,26)(7,19)(8,21,23,21,32)(4,14,15,26)(7,19)(8,21,23,21,32)(4,14,15,26)(7,19)(8,21,23,21,32)(4,14,15,26)(7,19)(8,21,23,21,32)(4,14,15,26)(7,19)(8,21,23,21,32)(4,14,15,26)(7,19)(8,21,23,23,21)(1,23,24,31)(1,23,$ $(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(17,28)(18,29)(20,30)(23,31)(27,32), (1,2,6,10)(3,17,13,28)(4,20,15,30)(5,22,16,9)(7,24,19,11)(8,26,21,14)(12,32,25,27)(18,23,29,31), (1,3,6,13)(2,7,10,19)(4,23,15,31)(5,25,16,12)(8,27,21,32)(9,29,22,18)(11,26,24,14)(17,30,28,20)]) \cong Q32$

 $P_{9} = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(3,12,23,23)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22)(11,23,24,31)(17,27,28,32), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,10,22), (1,5,6,16)(2,9,12), (1,5,6,16)(2,9,12), (1,5,6,16)(2,9,12), (1,5,6,16)(2,9,12), (1,5,6,16)(2,9,12), (1,5,6,16)(2,9,12), (1,5,6,16)(2,9,12), (1,5,6,16)(2,9,12), (1,5,$ $P_{10} = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,32,25,27)(18,23,29,31)]) \cong Q16$ $P_{11} = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(4,4,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,13,25)(4,14,15,26)(7,18,19,29)(8,20,21,30)(11,23,24,31)(17,27,28,32), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,10,22)(3,12,23,23), (1,4,16,26,6,15,5,14)(2,9,12,23), (1,4,16,26,6,15,5,14)(2,9,12,23), (1,4,16,26,6,15,5,14)(2,9,12,23), (1,4,16,26,6,15,5,14)(2,9,12,23), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,30), (1,4,15,26)(11,23,24,31)(11,22,23,31)(11,22,23,31)(11,22,23,31)(11,22,23,31)(11,22,23,31)$ $P_{12} = Group([(1,6)(2,10)(3,13)(4,15)(5,16)(7,19)(8,21)(9,22)(11,24)(12,25)(14,26)(7,18)(9,22)(11,24)(12,32)(9,29,22,18)(11,25,31,13,24)(12,32)(9,29,22,18)(11,25,31,13,24)(12,32)(9,29,22,18)(11,25,31,13,24)(12,32)(9,29,22,18)(11,25,31,13,24)(12,32)(9,29,22,18)(11,25,31,13,24)(12,32)(9,29,22,18)(11,25,31,13,24)(12,32)(9,29,22,18)(11,25,31,13,24)(12,32)(11,25,31,13)(11,25,31,13)(11,25,31,13)(11,25,31,13)(11,$

-1

 $E(8) - E(8)^3$

 $-E(8) + E(8)^3$

 $-E(16) + E(16)^7$

0

_1

 $-E(8) + E(8)^3$

 $E(8) - E(8)^3$

 $0 \quad E(16)^3 - E(16)^5$

-1

 $-E(8) + E(8)^3$

 $E(8) - E(8)^3$

-2

 $-E(8) + E(8)^3 = 0 - E(16)^3 + E(16)^5$

 $E(8) - E(8)^3 = 0 = E(16) - E(16)^7$

 $E(8) - E(8)^3 = 0 - E(16) + E(16)^7$

 $0 \quad -E(16)^3 + E(16)^5 \quad -E(8) + E(8)^3 \quad 0 \quad E(16)^3 - E(16)^5$